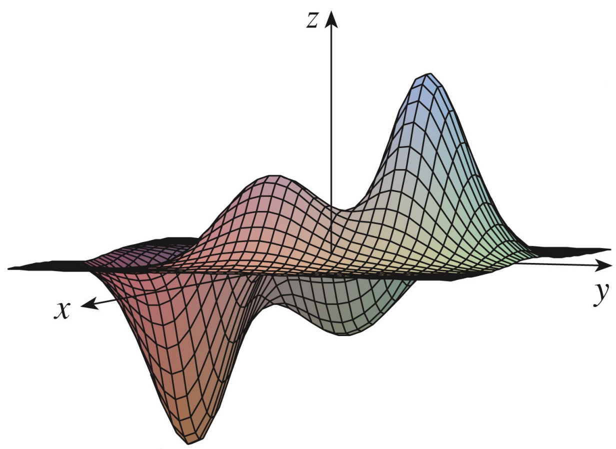
 

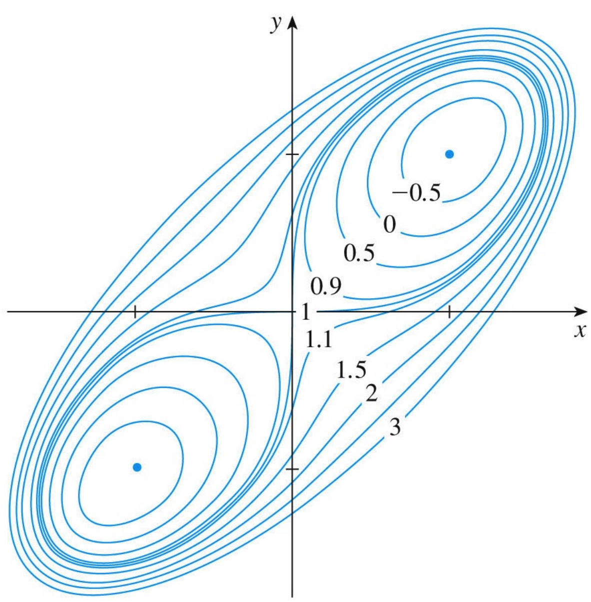
# Lesson 25. Local Minima and Maxima

## 1 Local minima and maxima

- Let $f$ be a function of two variables
- $f$ has a **local maximum** at $(a, b)$ if $f(a, b) \geq f(x, y)$ for all $(x, y)$ "close" to $(a, b)$
- $f$ has a **local minimum** at $(a, b)$ if $f(a, b) \leq f(x, y)$ for all $(x, y)$ "close" to $(a, b)$




**Example 1.** The contour map for $f(x, y) = x^4 + y^4 - 4xy + 1$ is shown below. Find the local maxima and minima of $f$.



## 2 Critical points: how to find local minima and maxima

- $(a, b)$ is a **critical point** of $f$ if

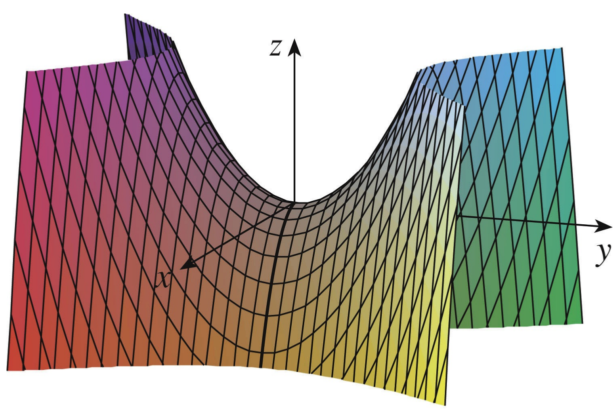
  or if one of these partial derivatives does not exist

- If $f$ has a local minimum or maximum at $(a, b)$, then $(a, b)$ is a critical point

- Finding local minima and maxima of $f$:

  1. Find all critical points of $f$
  2. Categorize each critical point using the **second derivatives test**:
     - Let $D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$
     - If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a
     - If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a
     - If $D(a, b) < 0$, then $(a, b)$ is a  of $f$
     - If $D(a, b) = 0$, the test gives no information

- Saddle points
  - Highest point in one direction, lowest point in the other direction
  - Graphically:




  - Saddle points look like hyperbolas in contour maps (see $(0, 0)$ in Example 1)

**Example 2.** Find the local minimum and maximum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$.

**Example 3.** Find the local minimum and maximum values and saddle points of $f(x, y) = y(e^x - 1)$.